Epicycles, Ellipses, & Apparent Motion: Ptolemaic, Copernican, Keplerian Models

This work was motivated by viewing *Majestic Clockwork: How we Discovered our Solar System*

This simulation of classical models of planetary motion was constructed by comparing geocentric epicycle representations and heliocentric Copernican and Keplerian orbits of Venus, Mercury, and Jupiter. Using simple trigonometric formulation, ***the apparent motion of each planet as seen from Earth*** is built up from combinations of uniform circular motions, following both the Copernican kinematic construction and the historical Ptolemaic epicycle framework. For Venus, these models naturally reproduce retrograde motion with relatively simple geometry, while for Mercury they reveal increasing complexity, requiring additional epicycles to approximate observed behavior. See Plots Below: Ptolemaic Mercury Multi Epicycle. This program then contrasts these geometrically elaborate constructions with Kepler's elliptical orbits, **showing how a single heliocentric ellipse replaces multiple epicycles** while yielding the same apparent geocentric phenomena. Together, the models illustrate how retrograde motion arises as a projection effect of relative motion, and why Mercury played a central role in exposing the limitations of the epicycle system.

Conceptual difference

Ptolemy and Copernicus both used circles, but they used them in fundamentally different ways.

In the Ptolemaic model, **Earth is fixed at the center.** Each planet moves on an epicycle, **whose center itself moves on a larger circle called the deferent**. direction" is a geocentric projection effect even inside Ptolemaic kinematics.
"Retrograde occurs when the epicycle motion temporarily dominates the deferent motion in the geocentric longitude."

In the Copernican model, **planets move around the Sun, not Earth**. Retrograde motion is not real motion at all, but an apparent effect that arises when Earth's heliocentric motion is subtracted from another planet's heliocentric motion

Mathematically, Ptolemy adds circles; Copernicus subtracts orbits.

The difference in equations

A. Ptolemaic-Style single-epicycle model (Earth fixed)

Earth is at the origin and never moves.

- R_0: deferent radius R
- R_1: epicycle radius
- ω_D: deferent angular speed
- ω_e: epicycle angular speed

✔ Retrograde motion comes from epicycle geometry.
✔ Earth never moves

Deferent angular speed (ω_D)

is the constant angular rate at which the center of a planet's epicycle moves around Earth along the deferent circle in a Ptolemaic (P) geocentric model.

$$x_P(t) = R_0\cos(\omega_D t) + R_1\cos(\omega_e t)$$
$$y_P(t) = R_0\sin(\omega_D t) + R_1\sin(\omega_e t)$$

Copernican model (heliocentric subtraction)

Earth and Venus both orbit the Sun.
Copernican (C) Heliocentric orbits:
ω_E: Earth heliocentric speed
ω_M: Mercury heliocentric speed

$$x_C(t) = x_V(t) - x_E(t)$$
$$y_C(t) = y_V(t) - y_E(t)$$

Apparent geocentric motion
✔ Retrograde motion arises from relative motion
✔ No physical epicycles exist

In the Copernican circular model, all planetary orbits share the same mathematical form; differences in apparent motion arise solely from orbital radius and period, not from different equations.

Circular Motion of Earth and Venus

$$x_E(t) = a_E\cos(\omega_E t), \quad y_E(t) = a_E\sin(\omega_E t)$$
$$x_V(t) = a_V\cos(\omega_V t), \quad y_V(t) = a_V\sin(\omega_V t)$$

In the Ptolemaic (P) System 150 AD

Retrograde motion is produced by epicycles added to a fixed Earth, whereas

In the Copernican (C) System 1543 AD

the same phenomenon emerges naturally from subtracting Earth's heliocentric motion from that of another planet.

In the Kepler (K) System 1619 AD

The orbits are plotted with ellipses

Ptolemaic, Copernican, and Kepler Models — Venus & Mercury

Except where noted, this work was done using the Mathcad Programming Language.

Clock: t_{max} = 2000 Days $\quad t_{max} := 2000 \quad N := 5000 \quad n := 0, 1 .. 2000 \quad t := 0, \frac{t_{max}}{N} .. t_{max}$

$T_E := 365.256 \quad T_V := 224.701 \quad T_M := 87.969 \quad X_E(t) := 0 \quad Y_E(t) := 0$

I. PTOLEMAIC (TRUE GEOCENTRIC)

Earth fixed. Retrograde from epicycles

Venus (single epicycle), Ptolemaic-style)

Ptolemaic Multi-epicycle Model of Mercury's irregular retrograde behavior played a decisive historical role by revealing the practical limits of epicycles and motivating Kepler's elliptical reformulation.

$\omega_{DV} := \frac{2 \cdot \pi}{T_E} \quad \omega_{EV} := \frac{2 \cdot \pi}{T_V} \quad R_{0V} := 1 \quad R_{1V} := 0.72$ Distance Units are in Scaled AU

$xPV(t) := R_{0V} \cdot \cos(\omega_{DV} \cdot t) + R_{1V} \cdot \cos(\omega_{EV} \cdot t) \qquad yPV(t) := R_{0V} \cdot \sin(\omega_{DV} \cdot t) + R_{1V} \cdot \sin(\omega_{EV} \cdot t)$

Mercury (multi-epicycle, Ptolemaic-style) - Illustrative, not Ptolemy's exact parameters

$\omega_{DM} := \frac{2 \cdot \pi}{T_E} \quad \omega_{E1} := \frac{2 \cdot \pi}{T_M} \quad \omega_{E2} := 2.6 \cdot \omega_{E1} \quad R_{0M} := 1 \quad R_{1M} := 0.38 \quad R_{2M} := 0.12$

$xPM(t) := R_{0M} \cdot \cos(\omega_{DM} \cdot t) + R_{1M} \cdot \cos(\omega_{E1} \cdot t) + R_{2M} \cdot \cos(\omega_{E2} \cdot t)$

$yPM(t) := R_{0M} \cdot \sin(\omega_{DM} \cdot t) + R_{1M} \cdot \sin(\omega_{E1} \cdot t) + R_{2M} \cdot \sin(\omega_{E2} \cdot t)$

II. HELIOCENTRIC COPERNICAN (CIRCULAR, RELATIVE MOTION)

$a_E := 1 \quad a_V := 0.723 \quad a_M := 0.387 \quad \omega_E := \frac{2 \cdot \pi}{T_E} \quad \omega_V := \frac{2 \cdot \pi}{T_V} \quad \omega_M := \frac{2 \cdot \pi}{T_M}$

$xE(t) := a_E \cdot \cos(\omega_E \cdot t) \qquad yE(t) := a_E \cdot \sin(\omega_E \cdot t)$

$xV(t) := a_V \cdot \cos(\omega_V \cdot t) \qquad yV(t) := a_V \cdot \sin(\omega_V \cdot t)$

$xM(t) := a_M \cdot \cos(\omega_M \cdot t) \qquad yM(t) := a_M \cdot \sin(\omega_M \cdot t)$

$xMC(t) := xM(t) - xE(t) \qquad yMC(t) := yM(t) - yE(t)$

$xVC(t) := xV(t) - xE(t) \qquad yVC(t) := yV(t) - yE(t)$

III. KEPLERIAN (ELLIPTICAL)

$e_E := 0.0167 \quad e_V := 0.0068 \quad e_M := 0.2056 \quad ME(t) := \frac{2 \cdot \pi \cdot t}{T_E} \quad MV(t) := \frac{2 \cdot \pi \cdot t}{T_V} \quad MM(t) := \frac{2 \cdot \pi \cdot t}{T_M}$

$EE(t) := ME(t) + e_E \cdot \sin(ME(t)) \quad EV(t) := MV(t) + e_V \cdot \sin(MV(t)) \quad EM(t) := MM(t) + e_M \cdot \sin(MM(t))$

$b_E := a_E \cdot \sqrt{1 - e_E^2} \quad b_V := a_V \cdot \sqrt{1 - e_V^2} \quad b_M := a_M \cdot \sqrt{1 - e_M^2}$

$xEK(t) := a_E \cdot (\cos(EE(t)) - e_E) \qquad yEK(t) := b_E \cdot \sin(EE(t))$

$xVK(t) := a_V \cdot (\cos(EV(t)) - e_V) \qquad yVK(t) := b_V \cdot \sin(EV(t))$

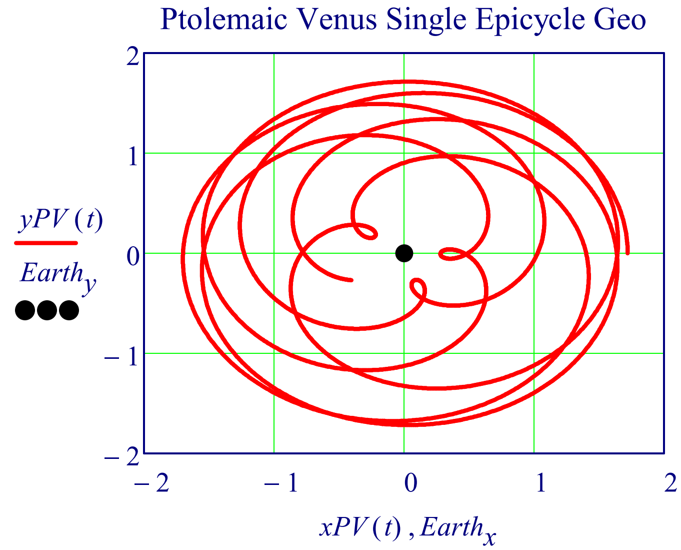
$xMK(t) := a_M \cdot (\cos(EM(t)) - e_M) \qquad yMK(t) := b_M \cdot \sin(EM(t))$

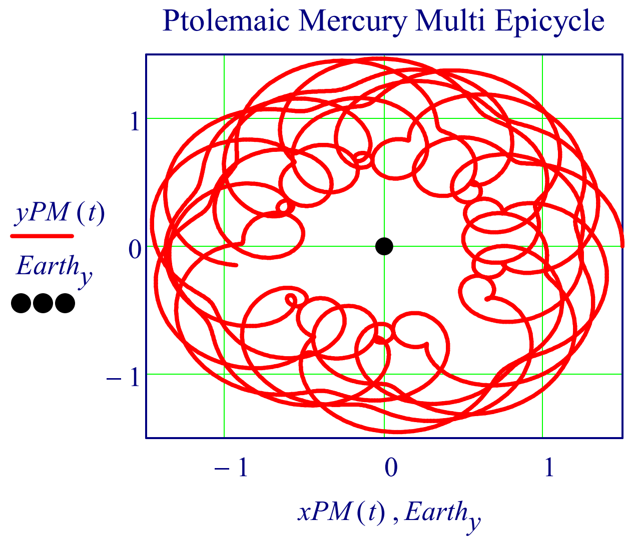
$xVKg(t) := xVK(t) - xEK(t) \qquad yVKg(t) := yVK(t) - yEK(t)$

$xMKg(t) := xMK(t) - xEK(t) \qquad yMKg(t) := yMK(t) - yEK(t)$

Note: Kepler solver is first-order; Mercury will have a small phase error.

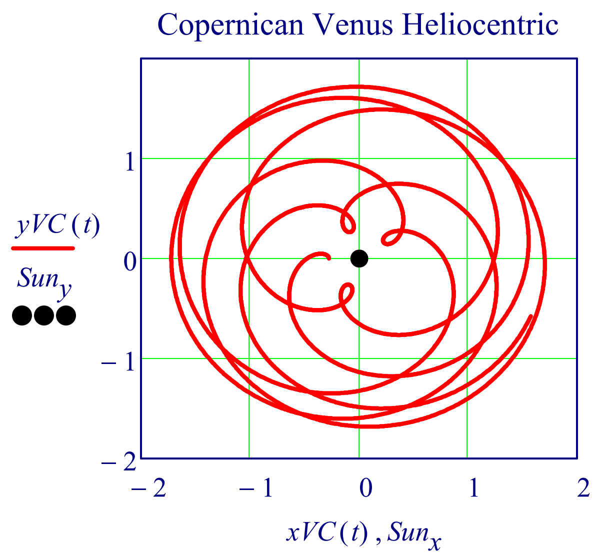
Time span shown: t = 0 … tmax (days). Apparent patterns repeat on synodic periods.

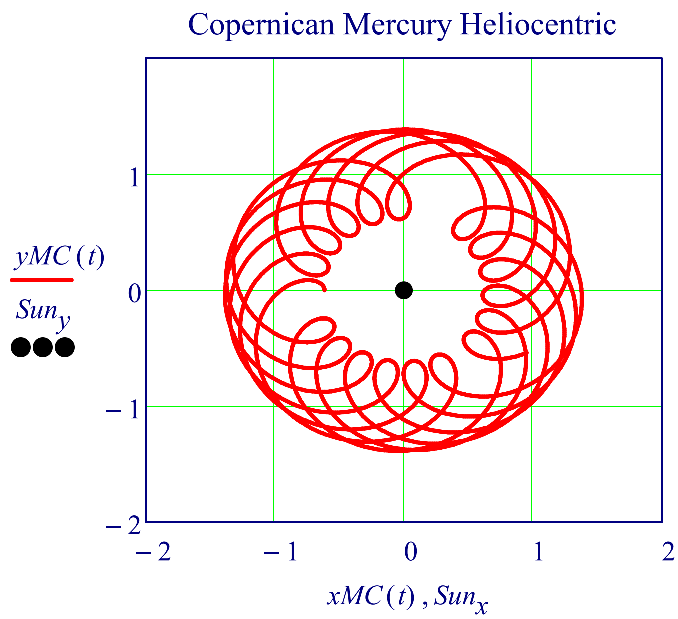


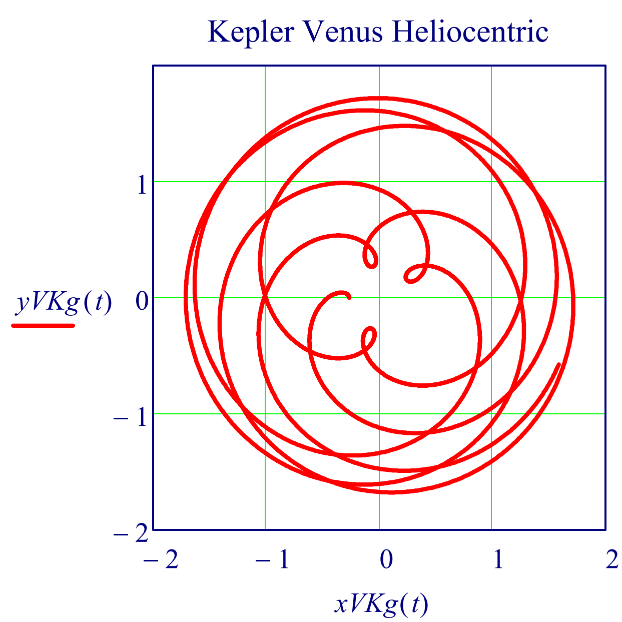


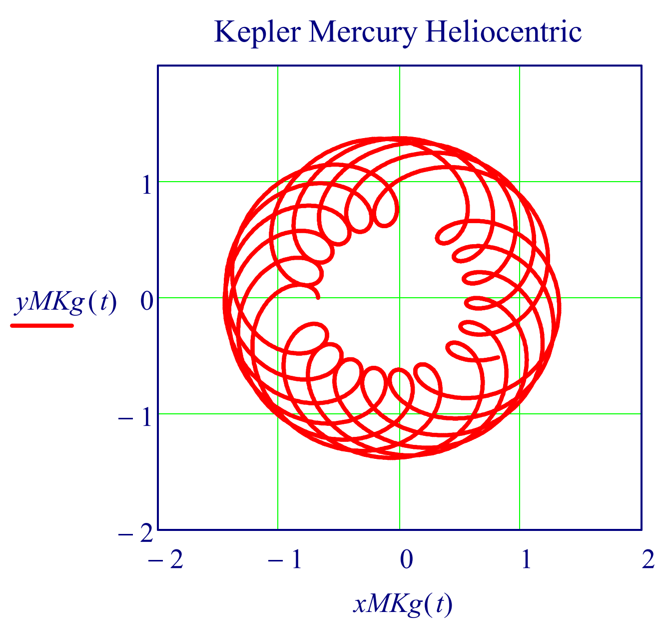
Above: Multi-epicycle fit produces drifting loops (non-closure) without additional parameters.

Copernican Venus — Geocentric Appearance (from heliocentric orbits









IV. DISTANCE-VS-TIME

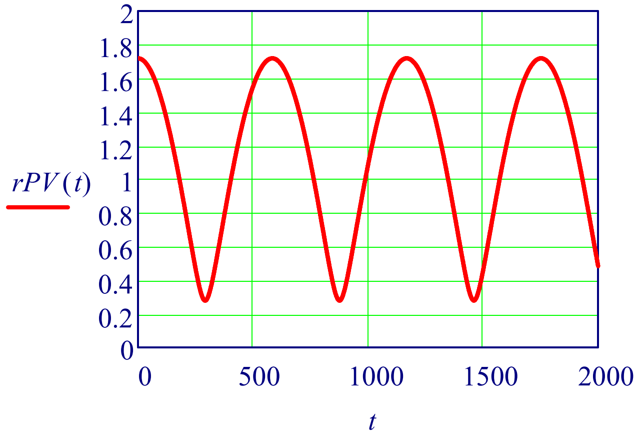
$$rPV(t) := \sqrt{xPV(t)^2 + yPV(t)^2} \qquad rPM(t) := \sqrt{xPM(t)^2 + yPM(t)^2}$$

$$rVC(t) := \sqrt{xVC(t)^2 + yVC(t)^2} \qquad rMC(t) := \sqrt{xMC(t)^2 + yMC(t)^2}$$

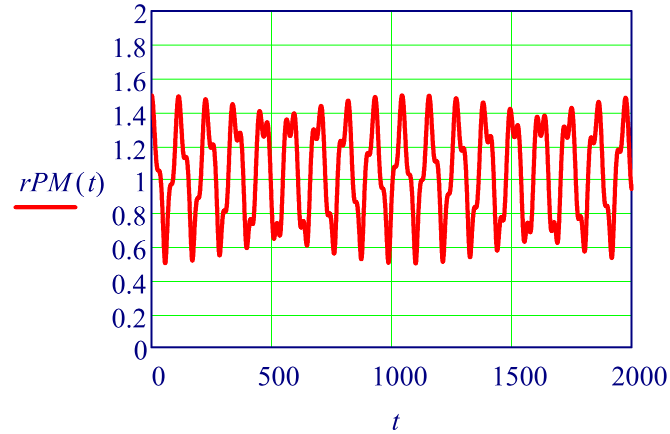
$$rVK(t) := \sqrt{xVKg(t)^2 + yVKg(t)^2} \qquad rMK(t) := \sqrt{xMKg(t)^2 + yMKg(t)^2}$$

Although this multi-epicycle Ptolemaic construction reproduces retrograde loops, the curve does not close after one synodic period: successive retrograde loops drift in phase and shape, indicating that a finite epicycle fit cannot simultaneously match Mercury's motion over all times without adding further ad-hoc geometric correction

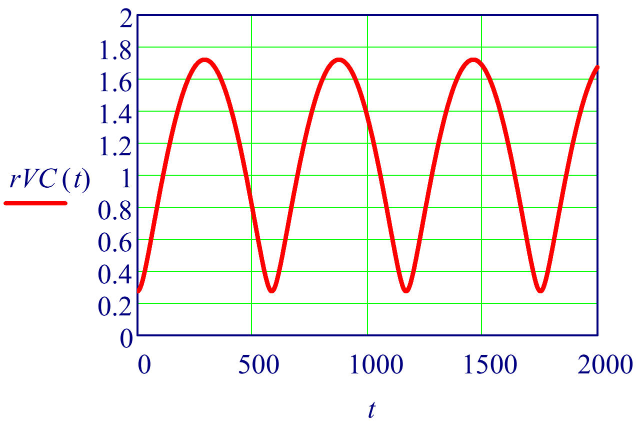
Ptoloemaic Venus Synodic Cycle



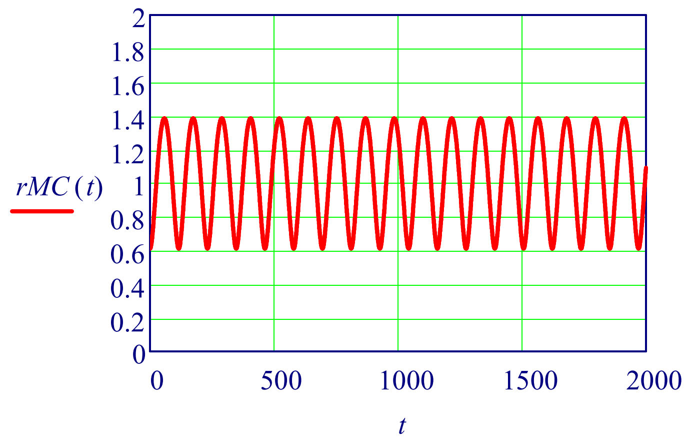
Ptolemaic Mercury Synodic Cycle



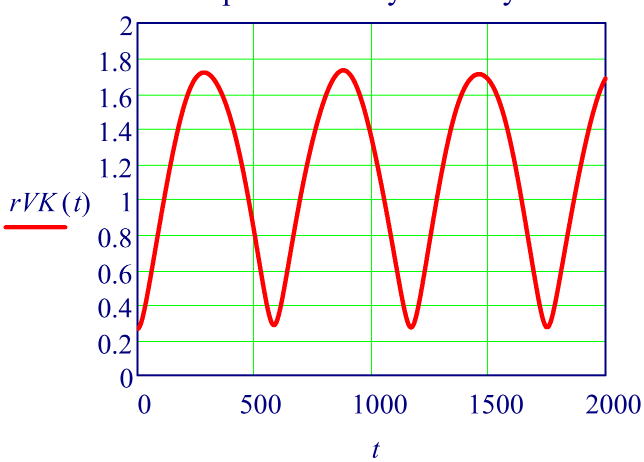
Copernicus Venus Synodic Cycle



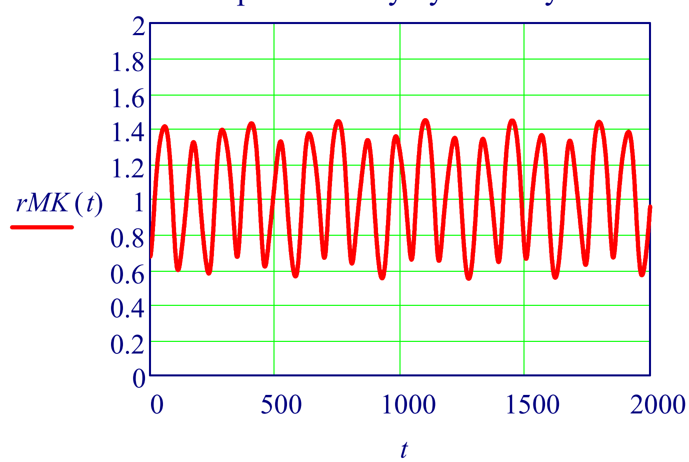
Copernicus Mercury Synodic Cycle



Kepler Venus Synodic Cycle



Kepler Mercury Synodic Cycle



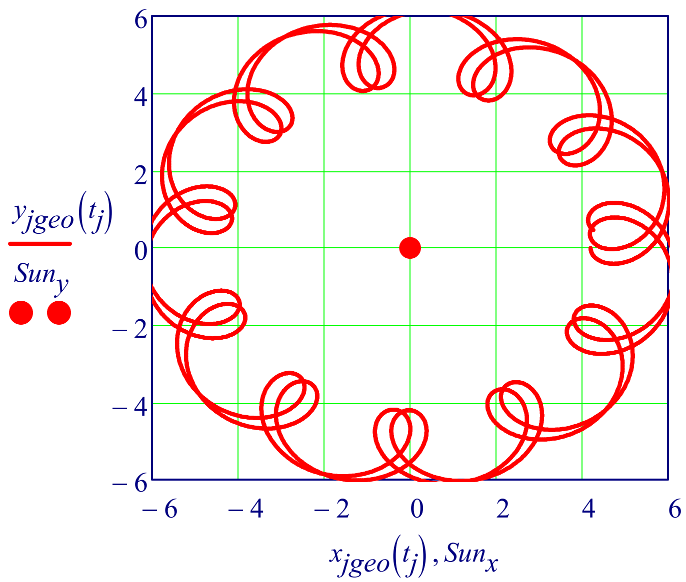
Heliocentric Orbit for Jupiter

Clock: t_{max} = 2 Jupiter Year $\quad t_{max} := 8800 \quad a_J := 5.20 \quad T_J := 4332.6 \quad t_j := 0, \frac{t_{max}}{N} .. t_{max}$

$$x_J(t_j) := a_J \cdot \cos\left(2\pi \cdot \frac{t_j}{T_J}\right) \qquad y_J(t_j) := a_J \cdot \sin\left(2\pi \cdot \frac{t_j}{T_J}\right)$$

$$x_{jgeo}(t_j) := x_J(t_j) - xE(t_j) \qquad y_{jgeo}(t_j) := y_J(t_j) - yE(t_j)$$

Copernican Jupiter Heliocentric



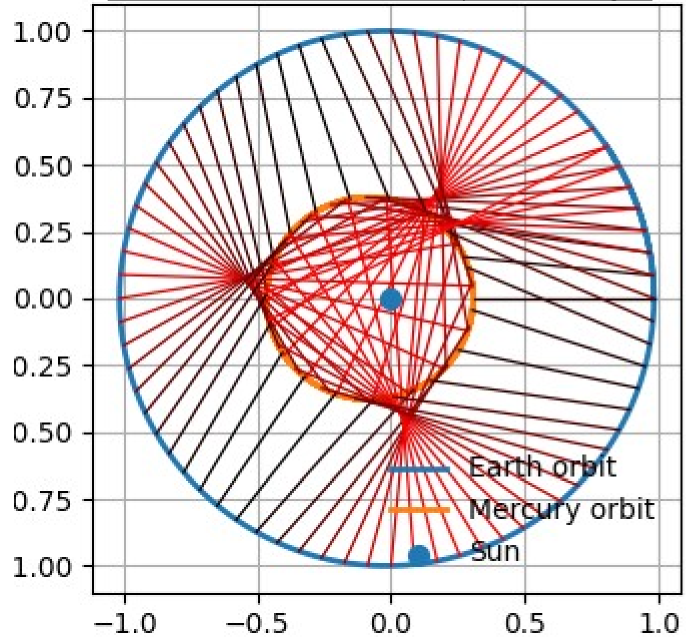
What men saw looking at Mercury: Rays of Light Seen from Earth to Mercury

Heliocentric Earth–Mercury lines of sight. Each ray represents the instantaneous visual line from Earth to Mercury and is drawn only between the observer and the planet, with no extension beyond Mercury's orbital position. Heliocentric construction of the Earth–Mercury distance with the Sun fixed at the origin. Straight rays represent the instantaneous difference vector between Earth and Mercury. The apparent complexity of Mercury's motion arises entirely from relative motion, without the need for epicycles

Mathcad's XY plots struggle with many disconnected segments; rays shown below were generated in Python.

View from Earth to Mercury Line-of-Sight

The outer blue ellipse is the orbit of the earth around the Mercury and the sun. The inner red ellipse is the orbit of Mercury. Heliocentric Earth–Mercury distance rays. Lines of sight are drawn from Earth to Mercury at uniform 5° intervals along Earth's orbit. This sampling emphasizes how Mercury's apparent direction changes as Earth progresses around the Sun.



Distance Encoded Scheme
Closest Mercury → black
Farthest Mercury → red